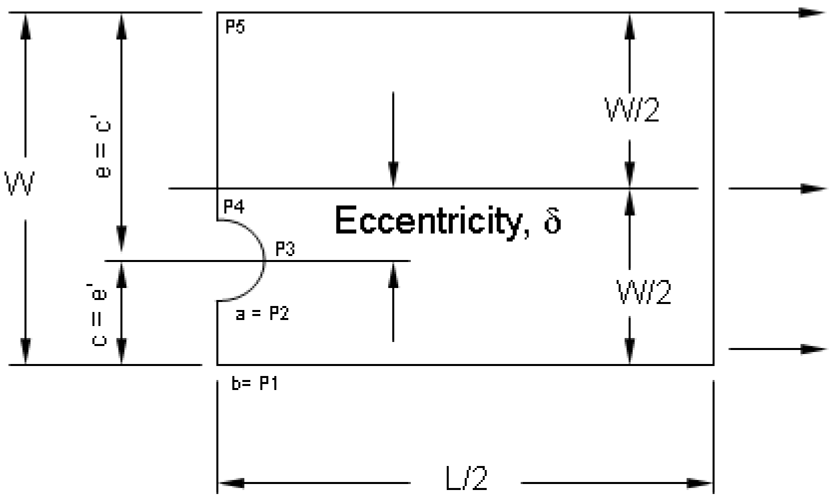
Nonlinear analytical solution based on linear maximum stress at a hole for 2014-T6 Aluminum Alloy

$\mathrm{ORIGIN} \equiv 1$

References
1. MMPDS-03, 1 October of 2006
2. ESDU 76016

Geometry of the hole: $P \equiv (-0.4375 \quad -0.1660 \quad -0.0880 \quad -0.0100 \quad 0.4375)^T \cdot in$



Far field traction

$\sigma_g \equiv 36.428 \cdot ksi$

Maximum Edge Distance: $e \equiv P_5 - P_3 = 0.525\,in$

Minimum Edge Distance: $c \equiv P_3 - P_1 = 0.349\,in$

Plate Width: $W := e + c = 0.875\,in$

Hole Diameter: $D \equiv P_4 - P_2 = 0.156\,in$

Hole Radius: $r \equiv \frac{D}{2} = 0.078\,in$

Nominal-Stress Path, from a to b: $a := r = 0.078\,in$

$b := c = 0.349\,in$

Dimensionless Hole location: $\psi := \frac{e}{c} = 1.504$

Dimensionless Edge short-edge Distance: $\lambda := \frac{D}{2 \cdot c} = 0.2232$

Hole-Location Eccentricity: $\delta := \left[\frac{\psi - 1}{2 \cdot (\psi + 1)}\right] \cdot W = 0.088\,in$

Stress-strain Curve for 2014-T6 Aluminum Extrusions for $t \geq 0.500 \cdot in$

Fundamental Properties at room temperature per Reference 1, page 3-50

Modulus of Elasticity: $E \equiv 10.8 \cdot 10^{3} \cdot ksi$

Ultimate Tensile Stress: $F_{tu} \equiv 64 \cdot ksi$

Yield Tensile Stress: $F_{ty} \equiv 58 \cdot ksi$

Ultimate Elongation: $e_{u} \equiv 7 \cdot \%$

Poisson's Ratio: $\mu := 0.33$ Only required for FEM solution

n = f(T): $n \equiv 26$ Reference 1, page 3-69

The following parameters are calculated in Appendix B

MMPDS Proportional Limit: $S_{PL} = 53.08 \cdot ksi$ $\varepsilon_{PL} = 0.512 \cdot \%$

$$\frac{S_{PL}}{S_{0.2}} = 91.5 \cdot \%$$

StressCheck Required: $S_{70E} = 58.327 \cdot ksi$ $\varepsilon_{70E} = 0.772 \cdot \%$

Notice that the ultimate strain, e_{u}, is measured after the test specimen has failed and there is no load. Therefore, the actual ultimate strain at the ultimate load, $F_{tu} = 64 \cdot ksi$, is:

$$\varepsilon_{u} = 7.59 \cdot \%$$

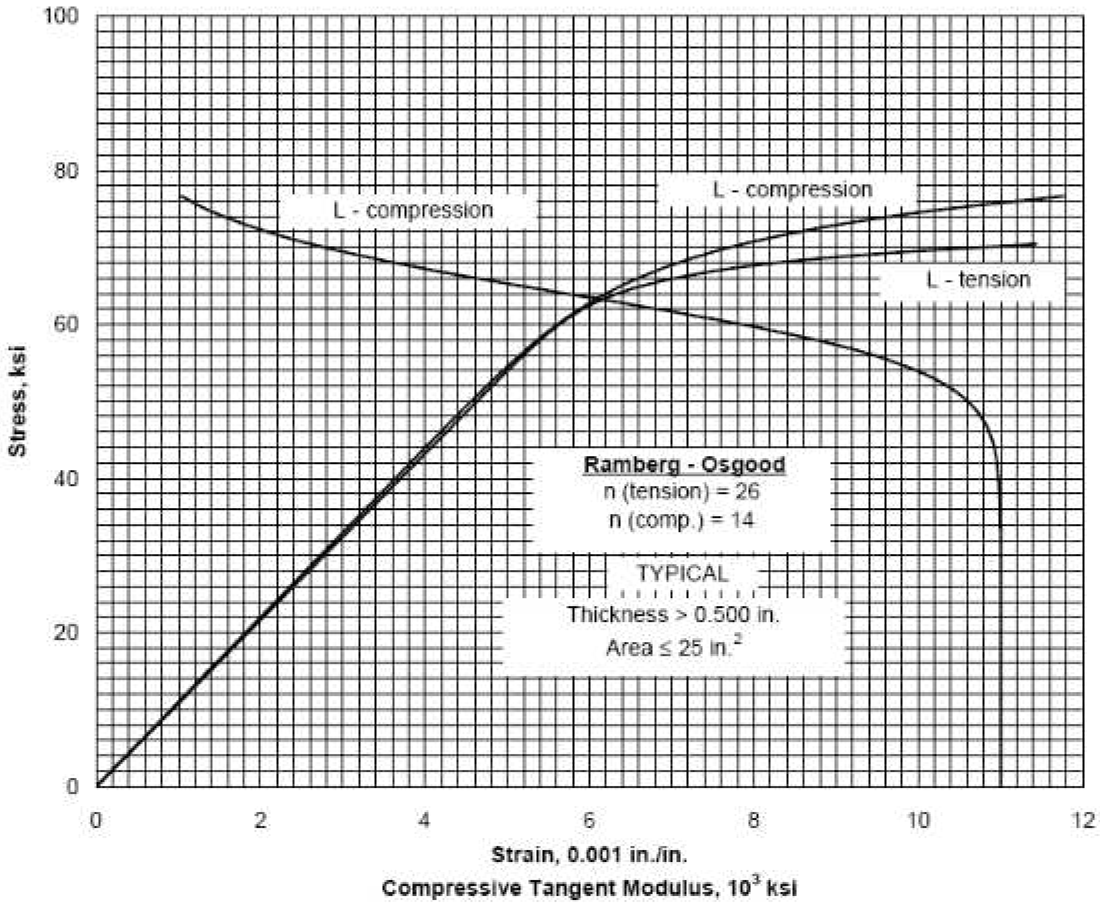


Figure 3.2.2.1.6(p). Typical tensile and compressive stress-strain and compressive tangent-modulus curves for 2014-T6 aluminum alloy extrusion at room temperature.

Figure 1: MMPDS-03 Stress Strain for Aluminum Alloy Extrusion 2014-T6

Perform a FEM study using the P-element formulation used in Stress Check for
$\psi = 1.504$ and the Hole-Location Eccentricity $\delta = 0.088\,\mathrm{in}$

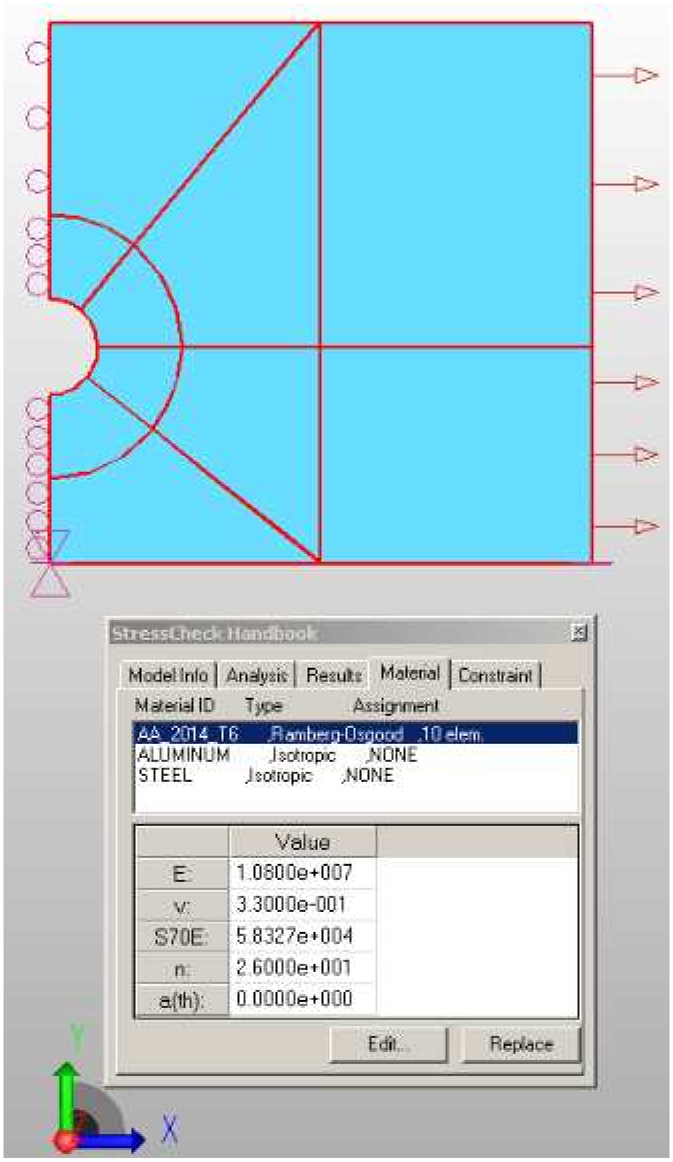


Figure 2: Material parameters StressCheck requires to generate the Ramberg-Osgood Stress-strain data set

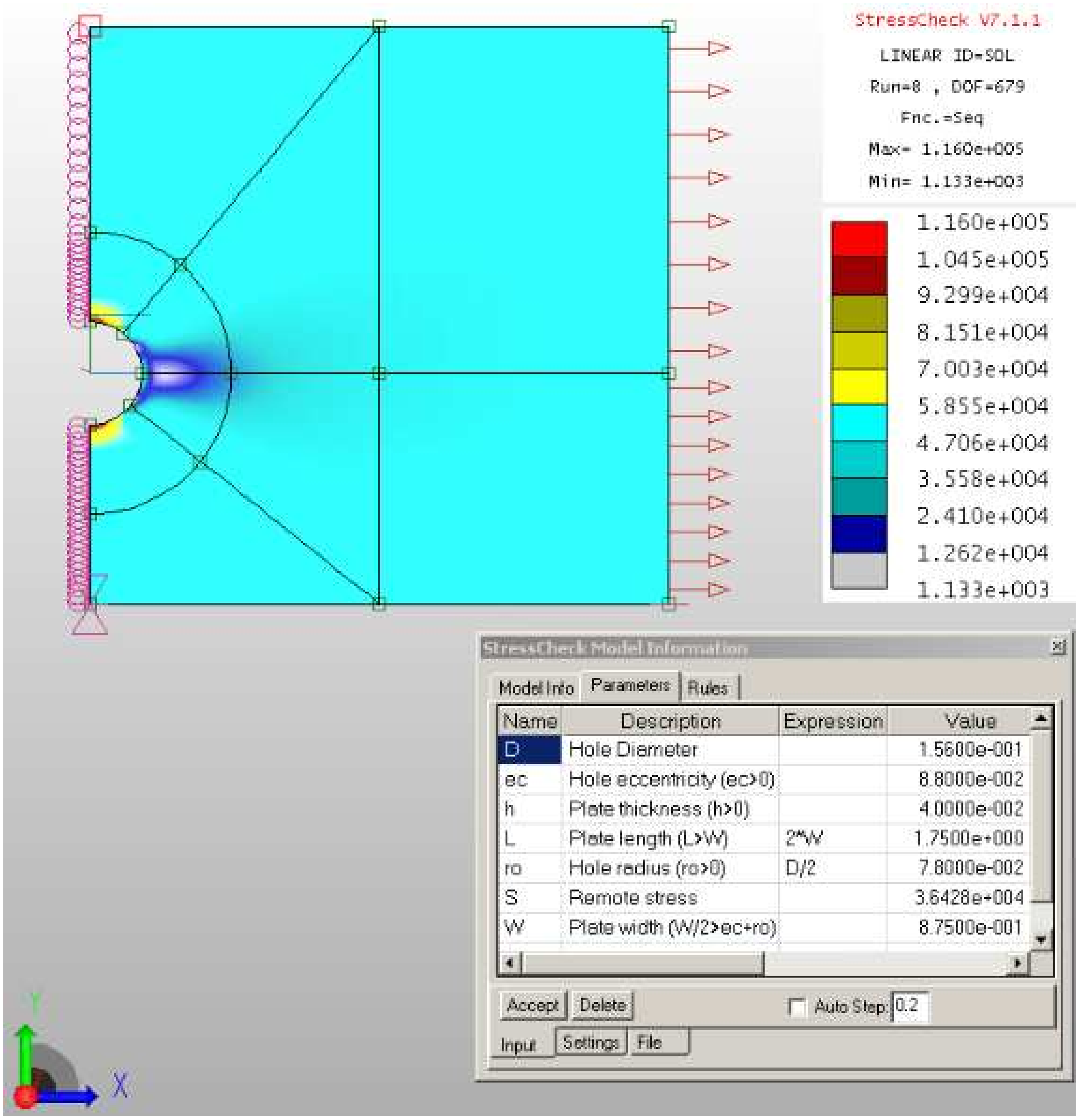


Figure 3: Linear stress solution using P-element StressCheck FEA Software

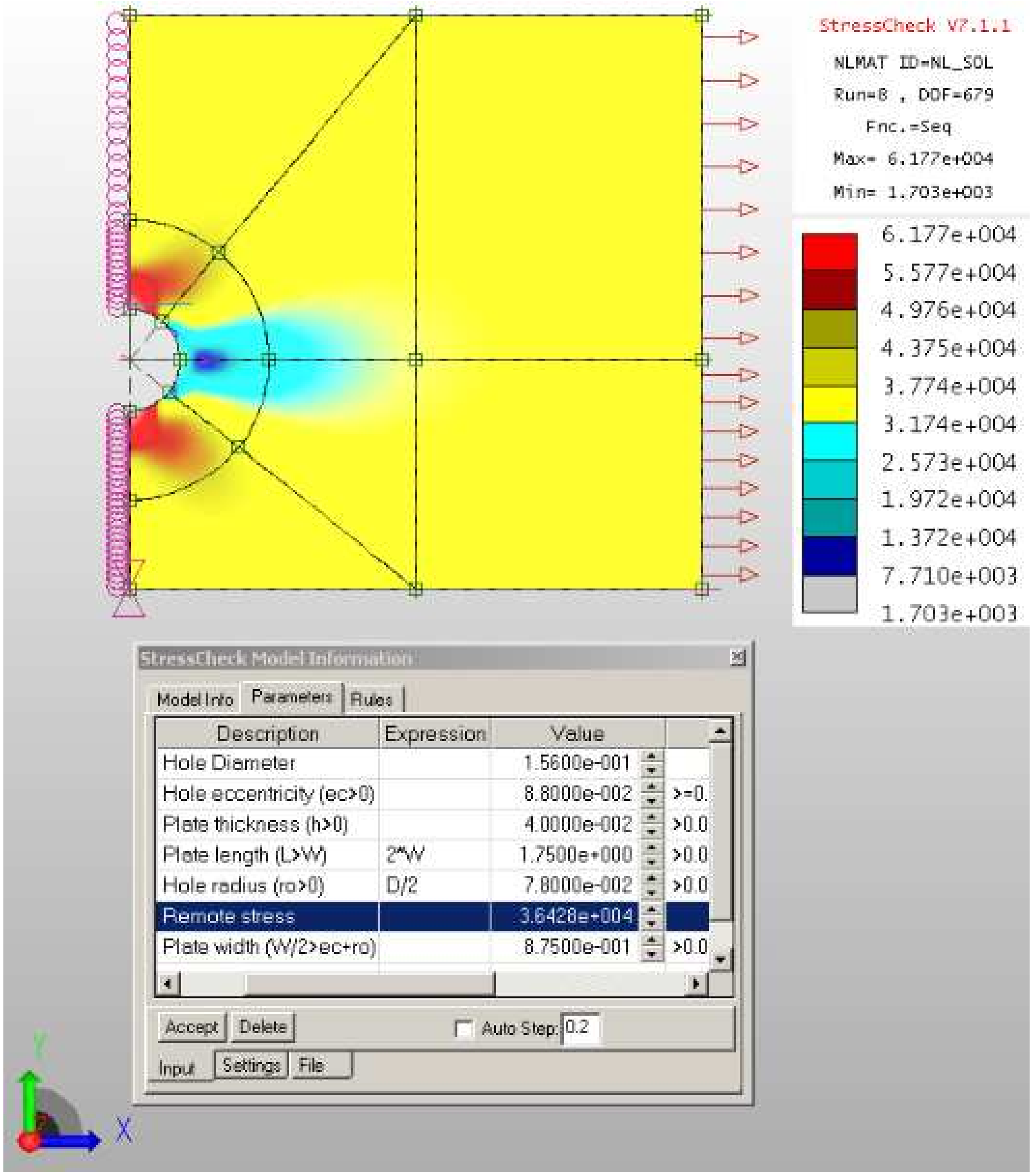


Figure 4: Nonlinear stress solution using P-element StressCheck FEA Software

FEM from StressCheck

Linear Solution | **Nonlinear Solution**

Maximum stress:

$\sigma_{Max} := \max(\sigma_{FEM}(Y)) = 116 \cdot ksi$ | $\sigma'_{Max} := \max(\sigma'_{FEM}(Y)) = 61.66 \cdot ksi$

Short-edge stress:

$$\sigma_{Nom} := \frac{\int_a^b \sigma_{FEM}(y)\,dy}{b-a} = 44.59 \cdot ksi \qquad \sigma'_{Nom} := \frac{\int_a^b \sigma'_{FEM}(y)\,dy}{b-a} = 44.05 \cdot ksi$$

Ultimate Stress:

$$\sigma_u := 1.50 \cdot \sigma_{Nom} = 66.88 \cdot ksi \qquad \sigma'_u := 1.50 \cdot \sigma'_{Nom} = 66.08 \cdot ksi$$

Gross SCF:

$$K_{tg\_FEM} := \frac{\sigma_{Max}}{\sigma_g} = 3.184 \qquad K'_{tg\_FEM} := \frac{\sigma'_{Max}}{\sigma_g} = 1.693$$

Nominal SCF:

$$K_{tn\_FEM} := \frac{\sigma_{Max}}{\sigma_{Nom}} = 2.602 \qquad K'_{tn\_FEM} := \frac{\sigma'_{Max}}{\sigma'_{Nom}} = 1.400$$

Nominal to gross transformation factor:

$$\Lambda_{FEM} := \frac{K_{tg\_FEM}}{K_{tn\_FEM}} = 1.2240 \qquad \Lambda'_{FEM} := \frac{K'_{tg\_FEM}}{K'_{tn\_FEM}} = 1.2093$$

$$\Lambda_{FEM} := \frac{\sigma_{Nom}}{\sigma_g} = 1.2240 \qquad \Lambda'_{FEM} := \frac{\sigma'_{Nom}}{\sigma_g} = 1.2093$$

$$MS_u := \frac{F_{tu}}{\sigma_u} - 1 = -0.04 \qquad MS'_u := \frac{F_{tu}}{\sigma'_u} - 1 = -0.03$$

Closed-form Solution

$\lambda = 0.2232$ $\quad K'_{tg}(\lambda,\psi) = 3.184$ $\quad \sigma_{Max\_CF} := K'_{tg}(\lambda,\psi) \cdot \sigma_g = 116.0 \cdot ksi$

$\psi = 1.504$ $\quad K'_{tn}(\lambda,\psi) = 2.481$

$$\Lambda_{CF} := \frac{K'_{tg}(\lambda,\psi)}{K'_{tn}(\lambda,\psi)} = 1.2830$$

or

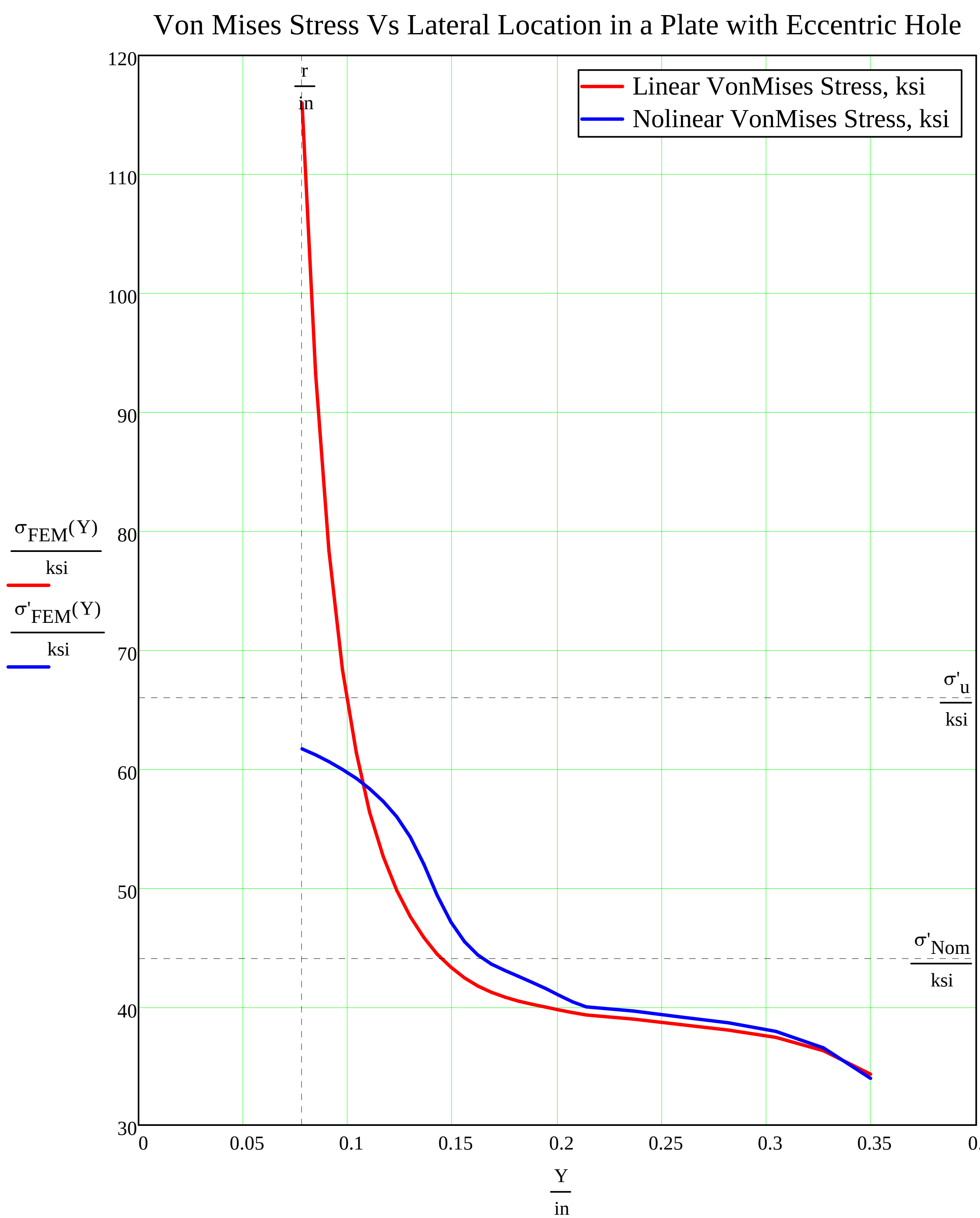
$$\Lambda(\lambda,\psi) = 1.276$$

The maximum strain found in Figure 5 is to be multiplied by 1.5 to obtain the ultimate strain to allow the calculation of the Margin of Safety based on strain rather than stress as follows

StressCheck FEM Maximum Nonlinear Strain: $\varepsilon_{Max\_SC} := 1.757 \cdot \%$

StressCheck Margin of Safety based on Strain:

$$MS_\varepsilon := \frac{e_u}{1.5 \cdot \varepsilon_{Max\_SC}} - 1 = 1.66$$


Von Mises Stress Vs Lateral Location in a Plate with Eccentric Hole
Linear VonMises Stress, ksi
Nolinear VonMises Stress, ksi
120
110
100
90
80
70
60
50
40
30
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
r/in
σFEM(Y)/ksi
σ'FEM(Y)/ksi
σ'u/ksi
σ'Nom/ksi
Y/in

Determine a nonlinear Margin of Safety based upon the maximum strain at the hole due to the negative Margin of Safety based on stress.

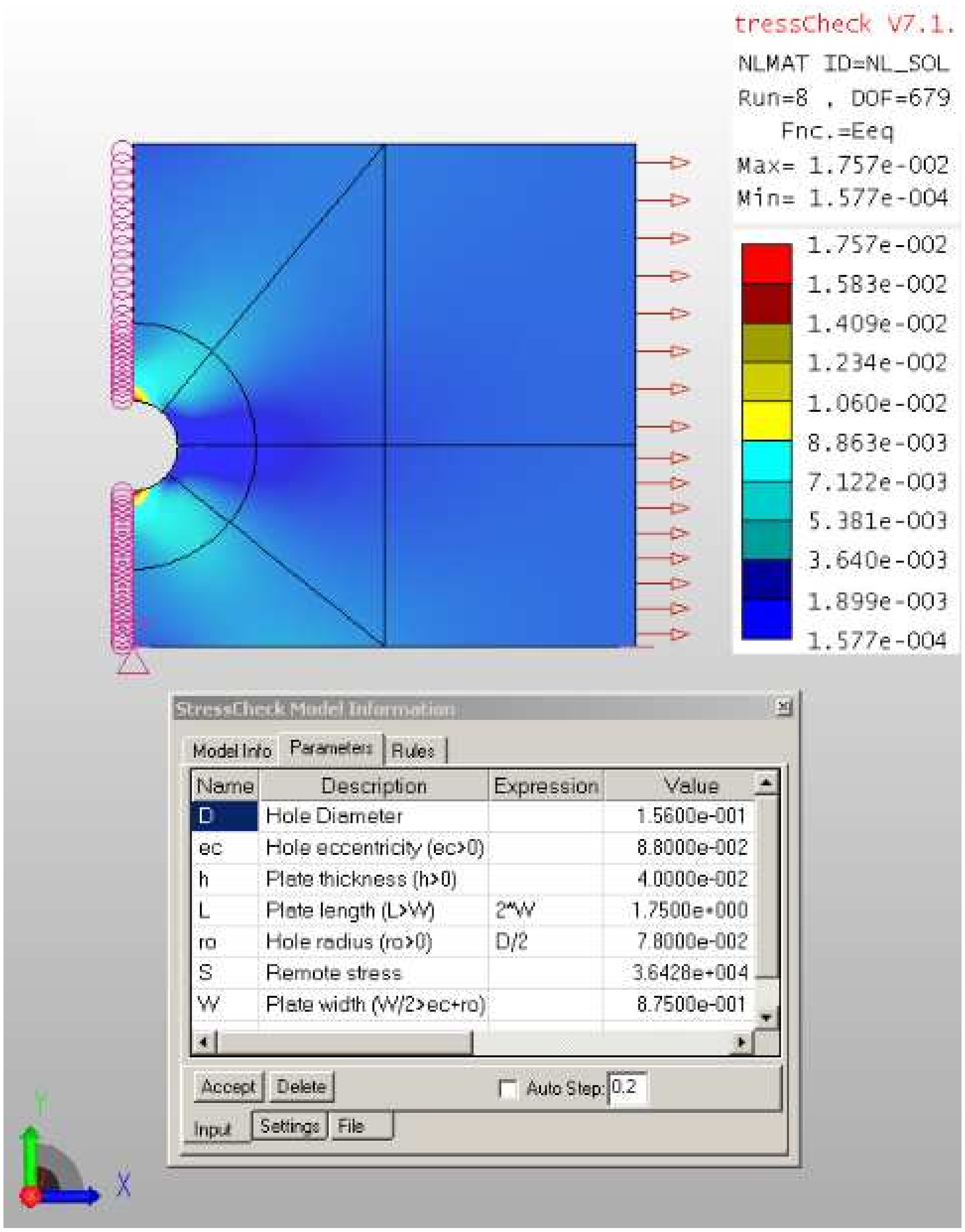
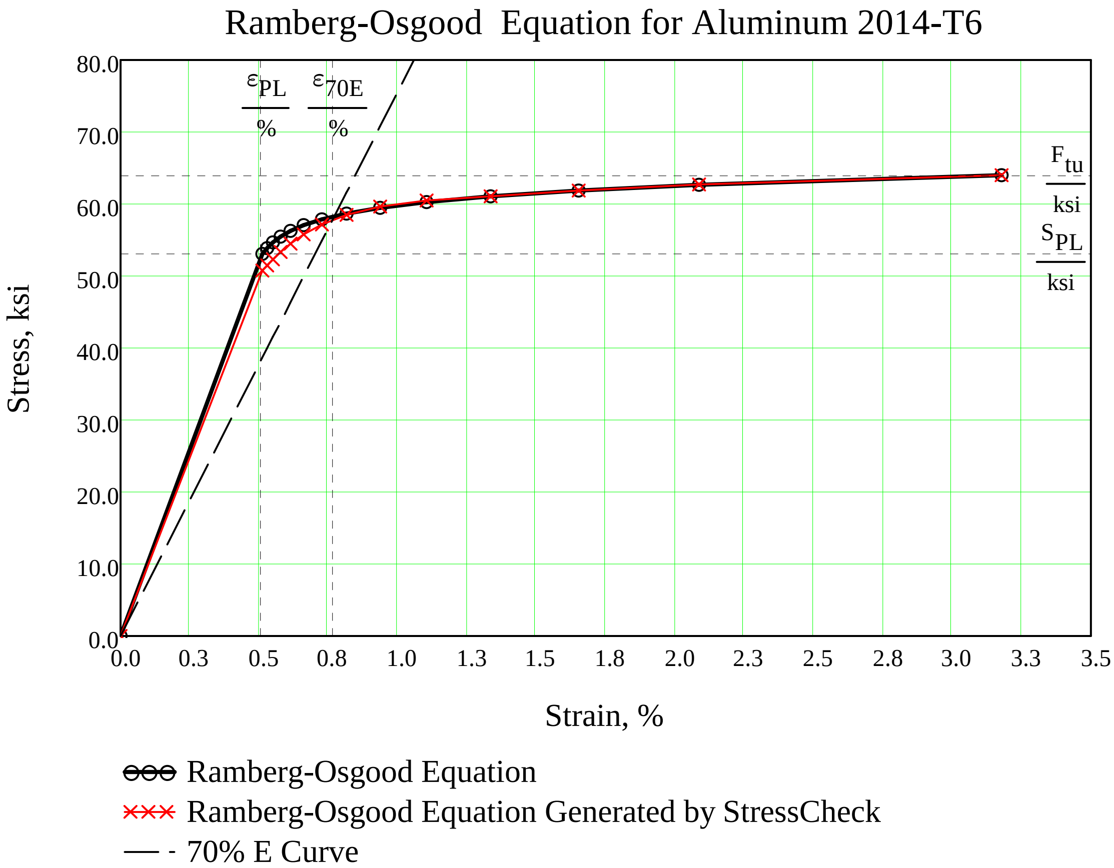


Figure 5: Nonlinear strain solution using P-element StressCheck FEA Software



MMPDS 03	**StressCheck**	**Relative Error**
$S_{PL} = 53.084 \cdot ksi$	$S'_{PL} := \sigma_{SC}(\varepsilon_{PL}) = 50.755 \cdot ksi$	$\%\Delta(S_{PL}, S'_{PL}) = -4.4 \cdot \%$
$S_{70E} = 58.327 \cdot ksi$	$S'_{70E} := \sigma_{SC}(\varepsilon_{70E}) = 57.918 \cdot ksi$	$\%\Delta(S_{70E}, S'_{70E}) = -0.7 \cdot \%$
$S_{u\_RO} = 70.774 \cdot ksi$	$S'_{u\_RO} := \sigma_{SC}(\varepsilon_u) = 66.491 \cdot ksi$	$\%\Delta(S_{u\_RO}, S'_{u\_RO}) = -6.1 \cdot \%$

Note

$\varepsilon_u = 7.593 \cdot \%$ is the strain obtained from Ramberg-Osgood equation Corresponding to the loaded test specimen. The actual ultimate strain provided in MMPDS (and similar reference books) is which is a measured strain of the ruptured (unloaded) test specimen

Apply the ESED (Equivalent Strain-Energy Density also known as Glinka) Method to obtain the pseudo nonlinear solution using the linear FEM maximum stress at the short-edge of a hole in a plate in a tensile stress-field.

Strain-energy constant: $\Gamma = \frac{1}{2} \cdot \varepsilon_L \cdot \sigma_{FEM}$ (1a)

$\sigma_{FEM} = E_0 \cdot \varepsilon_L$ (2)

Solve for the linear strain, ε_L from (2) as

$\varepsilon_L = \frac{\sigma_{FEM}}{E_0}$ (3)

Substitute (3) into (1) to get $\boxed{\Gamma = \frac{1}{2} \cdot \frac{\sigma_{FEM}^2}{E_0}}$ (1b)

Determine the strain at which the Ramberg-Osgood equates the strain-energy constant, Γ (Equation 1b)

The linear FEM Maximum Stress Using P-elements (StressCheck) is $\sigma_{FEM} \equiv 116 \cdot \text{ksi}$

The strain-energy for the Given σ_{FEM} is $\boxed{\Gamma := \frac{1}{2} \cdot \frac{\sigma_{FEM}^2}{E} = 622.96\,\text{psi}}$

Initial guess: $\varepsilon := \varepsilon_{0.2} = 0.737 \cdot \%$

Given

$$\Gamma = \int_0^{\varepsilon} \sigma_{RO\_root}(\varepsilon)\, d\varepsilon$$

$\varepsilon_G := \text{Find}(\varepsilon)$

The Pseudo Non-linear Strain is $\boxed{\varepsilon_G = 1.289 \cdot \%}$

Post-process

The Pseudo Non-linear Strain is $\boxed{\sigma_{Glinka} := \sigma_{RO\_root}(\varepsilon_G) = 65.466 \cdot ksi}$

The P-element (StressCheck) Maximum Non-linear Strain is $\varepsilon_{Max\_SC} = 1.757 \cdot \%$

The ESED (Glinka) Maximum Pseudo Non-linear Strain is $\varepsilon_G = 1.289 \cdot \%$

Relative Error of the ESED strain solution with respect to the P-element (FEM) solution is

$\%\Delta(\varepsilon_{Max\_SC}, \varepsilon_G) = -26.66 \cdot \%$

StressCheck Margin of Safety based on Strain: $\boxed{MS_{\varepsilon} := \frac{e_u}{1.5 \cdot \varepsilon_{Max\_SC}} - 1 = 1.66}$

ESED Margin of Safety based on Strain: $\boxed{MS_u := \frac{e_u}{1.50 \cdot \varepsilon_G} - 1 = 2.62}$

The P-element (StressCheck) Maximum Non-linear Stress is $\sigma'_{Max} = 61.66 \cdot ksi$

The ESED (Glinka) Maximum Pseudo Non-linear Stress is $\sigma_{Glinka} = 65.466 \cdot ksi$

Relative Error of the ESED stress solution with respect to the P-element (FEM) solution is

$\%\Delta(\sigma'_{Max}, \sigma_{Glinka}) = 6.17 \cdot \%$

Conclusion

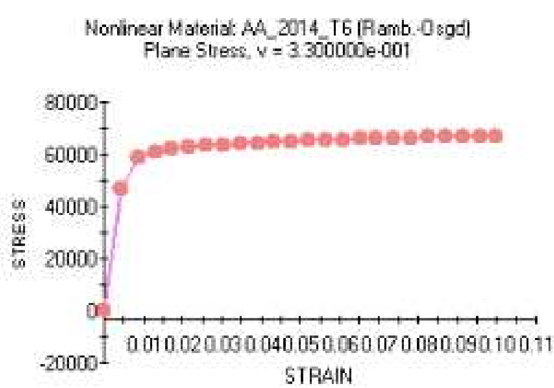
The ESED (Equivalent-strain Energy Density) also known as Glinka can be used to obtain an estimate of the plastic strain and corresponding MS (Margin of Safety) when the linear solution produces a negative MS indicating nonlinear material behavior due to high loads.

It should be noted that this approach will most likely occur for the ultimate load Margin of Safety, MS_u, rather than the limit load (yield) margin of safety, MS_Y

Appendix A: StressCheck P-element FEM Solution Data and Cubic Spline Procedure

Ramberg-Osgood Numerical Results from StressCheck

$$\mathrm{Data}_{SC} \equiv \begin{pmatrix} 0.0000000 & 0 \\ 0.0043478 & 46872 \\ 0.0086957 & 59076 \\ 0.0130430 & 60992 \\ 0.0173910 & 62066 \\ 0.0217390 & 62818 \\ 0.0260870 & 63397 \\ 0.0304350 & 63870 \\ 0.0347830 & 64269 \\ 0.0391300 & 64615 \\ 0.0434780 & 64920 \\ 0.0478260 & 65193 \\ 0.0521740 & 65440 \\ 0.0565220 & 65666 \\ 0.0608700 & 65875 \\ 0.0652170 & 66068 \\ 0.0695650 & 66248 \\ 0.0739130 & 66416 \\ 0.0782610 & 66575 \\ 0.0826090 & 66724 \\ 0.0869570 & 66866 \\ 0.0913040 & 67001 \\ 0.0956520 & 67129 \\ 0.1000000 & 67251 \end{pmatrix}$$



<u>Linear Solution Data</u>

$$\mathrm{Data} \equiv \begin{pmatrix} -0.1660 & 116000.0 \\ -0.1725 & 93230.0 \\ -0.1789 & 78340.0 \\ -0.1854 & 68310.0 \\ -0.1919 & 61340.0 \\ -0.1983 & 56320.0 \\ -0.2048 & 52600.0 \\ -0.2113 & 49750.0 \\ -0.2177 & 47540.0 \\ -0.2242 & 45780.0 \\ -0.2306 & 44380.0 \\ -0.2371 & 43270.0 \\ -0.2436 & 42390.0 \\ -0.2500 & 41700.0 \\ -0.2565 & 41170.0 \\ -0.2630 & 40760.0 \\ -0.2694 & 40430.0 \\ -0.2759 & 40160.0 \\ -0.2824 & 39920.0 \\ -0.2888 & 39690.0 \\ -0.2953 & 39470.0 \\ -0.3018 & 39265.0 \\ -0.3244 & 38910.0 \\ -0.3470 & 38450.0 \\ -0.3696 & 37990.0 \\ -0.3923 & 37370.0 \\ -0.4149 & 36260.0 \\ -0.4375 & 34270.0 \end{pmatrix}$$

<u>Nonlinear Solution Data</u>

$$\mathrm{Data'} \equiv \begin{pmatrix} -0.1660 & 61660 \\ -0.1725 & 61150 \\ -0.1789 & 60580 \\ -0.1854 & 59930 \\ -0.1919 & 59180 \\ -0.1983 & 58300 \\ -0.2048 & 57250 \\ -0.2113 & 55940 \\ -0.2177 & 54230 \\ -0.2242 & 51950 \\ -0.2306 & 49330 \\ -0.2371 & 47070 \\ -0.2436 & 45430 \\ -0.2500 & 44310 \\ -0.2565 & 43550 \\ -0.2630 & 42990 \\ -0.2694 & 42510 \\ -0.2759 & 42010 \\ -0.2824 & 41480 \\ -0.2888 & 40920 \\ -0.2953 & 40370 \\ -0.3018 & 39935 \\ -0.3141 & 39790 \\ -0.3264 & 39540 \\ -0.3388 & 39270 \\ -0.3511 & 39000 \\ -0.3635 & 38740 \\ -0.3758 & 38440 \\ -0.3881 & 38040 \\ -0.4005 & 37480 \\ -0.4128 & 36660 \\ -0.4252 & 35510 \\ -0.4375 & 33920 \end{pmatrix}$$

$$F_{CS}(x,X,Y) \equiv \left| \begin{array}{l} \text{"Cubic-spline"} \\ S \leftarrow \text{cspline}(X,Y) \\ y \leftarrow \text{interp}(S,X,Y,x) \end{array} \right.$$

StressCheck Calculated Ramberg-Osgood Material Stress-strain Data

$$\varepsilon_{SC\_RO\_Data} \equiv Data_{SC}^{\langle 1 \rangle}$$

$$\sigma_{SC\_RO\_Data} \equiv Data_{SC}^{\langle 2 \rangle} \cdot psi$$

$$\sigma_{SC}(\varepsilon) \equiv F_{CS}\left(\varepsilon, \varepsilon_{SC\_RO\_Data}, \sigma_{SC\_RO\_Data}\right)$$

StressCheck Linear solution P-elements

$$Y \equiv \left(P_3 - Data^{\langle 1 \rangle} \cdot in\right)$$

$$S_{SC} \equiv Data^{\langle 2 \rangle} \cdot psi$$

$$\sigma_{FEM}(y) \equiv F_{CS}\left(y, Y, S_{SC}\right)$$

StressCheckNonlinear solution P-elements

$$Y' \equiv \left(P_3 - Data'^{\langle 1 \rangle} \cdot in\right)$$

$$S'_{SC} \equiv Data'^{\langle 2 \rangle} \cdot psi$$

$$\sigma'_{FEM}(y) \equiv F_{CS}\left(y, Y', S'_{SC}\right)$$

Appendix B: Functions

Ramberg-Osgood Stress-strain equation:

The defined 0.2% offset strain is: $e_{0.2} \equiv 0.2\cdot\%$

Yield Stress: $S_{0.2} \equiv F_{ty} = 58\cdot ksi$

Yield Strain: $\boxed{\varepsilon_{0.2} \equiv e_{0.2} + \frac{S_{0.2}}{E} = 0.74\cdot\%}$

0.2% offset Linear-stress: $\sigma_L(\varepsilon) \equiv E\cdot(\varepsilon - e_{0.2})$

Normalization stress: $\sigma_n \equiv S_{0.2} = 58\cdot ksi$

$$\boxed{\varepsilon_{RO}(\sigma) \equiv \frac{\sigma}{E} + e_{0.2}\cdot\left(\frac{\sigma}{\sigma_n}\right)^n}$$

Equation [9.8.4.1.2(b)]
Ref. 1, page 9-200

Reference 1, Section **9.8.4.6.1 (page 9-200)** states:
If the proportional limit stress is equated with a plastic strain level of 0.0002 or a 0.02 percent deviation from linearity, and the Ramberg- Osgood relationship is found to be valid for small plastic strains, then the proportional limit stress, S_{PL} **=** f_{pl}, can be approximated from Equation 9.8.4.6(a) as follows:

Proportional Limit:

$$\boxed{S_{PL} \equiv S_{0.2}\cdot(0.1)^{\frac{1}{n}} = 53.08\cdot ksi}$$

Section [9.8.4.6.1]
Ref. 1, page 9-200

$$\boxed{\varepsilon_{PL} \equiv \varepsilon_{RO}(S_{PL}) = 0.512\cdot\%}$$

MMPDS 70%E Data point

$$\boxed{\varepsilon_{70E} \equiv \left[\left[150\cdot\left(\frac{S_{0.2}}{70\cdot\%\cdot E}\right)^n\right]^{\left[\frac{1}{(n-1)}\right]}\right] = 0.772\cdot\%}$$

$$\boxed{S_{70E} \equiv \left[\left[\frac{\left(\frac{S_{0.2}}{ksi}\right)}{\left(\frac{7}{3}\cdot e_{0.2}\cdot\frac{E}{ksi}\right)^{\frac{1}{n}}}\right]^{\frac{n}{(n-1)}}\cdot ksi\right] = 58.327\cdot ksi}$$

Define the 0.70xE straight line which when intercepting the Stress-strain curve will produce the S_{70E} stress level required by StressCheck as $\sigma_{70E}(\varepsilon) \equiv (0.70\cdot E)\cdot\varepsilon$

Ultimate Stress: $S_u \equiv F_{tu} = 64\cdot ksi$ $\boxed{\varepsilon_u \equiv e_u + \frac{F_{tu}}{E} = 7.59\cdot\%}$

Relative-Error: $\%\Delta(a,b) \equiv \left(\frac{b-a}{a}\right)$

The Ramberg-Osgood Equation **[9.8.4.1.2(b)]** is an implicit equation which needs to be solved using a root-finding procedure since $\varepsilon = f(\sigma)$, and not $\sigma = g(\varepsilon)$. However, there is an approximation, $\sigma_{ARO}(\varepsilon)$, given in ESDU 76016, page 76016 as equation 5.1 which can be used as an estimate of the stress, $\sigma_{RO}(\varepsilon)$, or even as a seed (starting value) for a more refined root-finding procedures.

$$\sigma_{ARO}(\varepsilon) \equiv \left| \begin{array}{l} \text{"ESDU 76016, page 76016, equation 5.1"} \\ k \leftarrow 0.79044\cdot n - 0.86977 \\ \beta \leftarrow \left(1 + \frac{1}{n}\right)^{(n-k-1)} - \left(1 + \frac{1}{n}\right)^{-k} \\ \lambda \leftarrow \left|\frac{\varepsilon\cdot E}{\sigma_n}\right| \\ \sigma_{EP} \leftarrow \varepsilon\cdot E\cdot\left(1 + \beta\cdot\lambda^{k} + \frac{1}{n}\cdot\lambda^{n-1}\right)^{\frac{-1}{n}} \end{array} \right.$$

$$E_t(\sigma) \equiv \frac{E}{1 + e_{0.2}\cdot n\cdot\frac{E}{\sigma_n}\cdot\left(\frac{\sigma}{\sigma_n}\right)^{n}}$$

[9.8.4.2(b)]

Ref. 1, page 9-205

$$E_t\left(S_{0.2}\right) = 1.011\times 10^{3}\cdot ksi$$

$$\sigma_{RO\_root}\left(\varepsilon_{Root}\right) \equiv \left| \begin{array}{l} \text{"Root-finding Procedure by Julio C. Banks, P.E."} \\ \text{"Using Newton's rather than MathCAD's Method"} \\ \varepsilon_1 \leftarrow \varepsilon_{Root} \\ \sigma_1 \leftarrow \sigma_{ARO}\left(\varepsilon_1\right) \\ \Delta\sigma_{Tol} \leftarrow 0.5\cdot psi \\ \Delta\sigma \leftarrow 2\cdot\Delta\sigma_{Tol} \\ i \leftarrow 1 \\ \text{while } \Delta\sigma > \Delta\sigma_{Tol} \\ \quad \left| \begin{array}{l} i \leftarrow i + 1 \\ \varepsilon_i \leftarrow \varepsilon_{RO}\left(\sigma_{i-1}\right) \\ \sigma_i \leftarrow \sigma_{i-1} - \left(\varepsilon_i - \varepsilon_{i-1}\right)\cdot E_t\left(\sigma_{i-1}\right) \\ \Delta\sigma \leftarrow \sigma_i - \sigma_{i-1} \end{array} \right. \\ \sigma \leftarrow \sigma_i \end{array} \right.$$

$$S_{u\_RO} \equiv \sigma_{RO\_root}\left(\varepsilon_u\right) = 70.774\cdot ksi$$

Create a procedure that generates a set of data points corresponding to the Ramberg-Osgood Equation (1) call such a set of discrete Data Points. Assume a linear stress up until the Proportional Limit.

$$
\text{RO\_Points} \equiv \left|
\begin{array}{l}
NP_e \leftarrow 2 \\
\varepsilon_1 \leftarrow 0\cdot\% \\
\sigma_1 \leftarrow 0\cdot ksi \\
\varepsilon_2 \leftarrow \varepsilon_{PL} \\
\sigma_2 \leftarrow S_{PL} \\
\Delta\sigma \leftarrow 0.8\cdot ksi \\
NP_p \leftarrow \text{round}\left(\dfrac{F_{tu} - S_{PL}}{\Delta\sigma}\right) \\
NP_{ep} \leftarrow NP_e + NP_p - 2 \\
\text{for } i \in 3..\left(NP_{ep}\right) \\
\quad \left| \begin{array}{l} \sigma_i \leftarrow \sigma_{i-1} + \Delta\sigma \\ \varepsilon_i \leftarrow \varepsilon_{RO}\left(\sigma_i\right) \end{array} \right. \\
\sigma_{NP_{ep}+1} \leftarrow F_{tu} \\
\varepsilon_{NP_{ep}+1} \leftarrow \varepsilon_{RO}\left(F_{tu}\right) \\
Data^{\langle 1\rangle} \leftarrow \varepsilon \\
Data^{\langle 2\rangle} \leftarrow \dfrac{\sigma}{ksi} \\
Data
\end{array}
\right.
$$

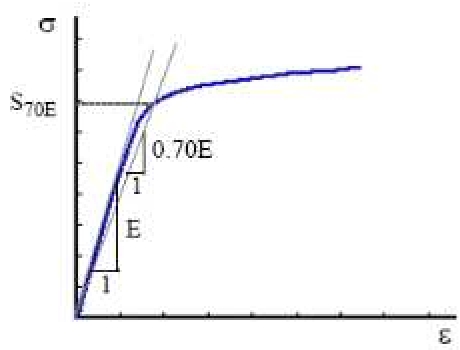
Ramberg-Osgood Discrete set of Data Points

$\varepsilon \equiv \text{RO\_Points}^{\langle 1\rangle}$ $\qquad$ $\sigma_{RO} \equiv \text{RO\_Points}^{\langle 2\rangle}\cdot ksi$

Number of points generated, $N_{RO} \equiv \text{length}(\varepsilon) = 15$

The Ultimate Ramberg-Osgood Strains is $\varepsilon_{u\_RO} \equiv \varepsilon_{N_{RO}} = 3.18\cdot\%$

Appendix C:
Determine the explicit equations for of the stress level corresponding to the interception point of the 0.70E line with the Ramberg-Osgood Curve



$$\boxed{\sigma_{70E}(\varepsilon) = (0.70\cdot E)\cdot\varepsilon} \qquad \text{(C1)}$$

Ramberg-Osgood: $\boxed{\varepsilon = \frac{\sigma}{E} + e_{0.2}\cdot\left(\frac{\sigma}{S_{0.2}}\right)^n}$

Equation [9.8.4.1.2(b)]
Ref. 1, page 9-200
Renamed here as (C2)

Where $n > 1$

$$e_{0.2} = 0.20\cdot\%$$

$$S_{0.2} = F_{ty}$$

Let the interception point be

$$\varepsilon = \varepsilon_{0.70E}$$

$$S_{0.70E} = \sigma_{70E}(\varepsilon_{0.70E})$$

Introduce the interception point definition into equations **C1** and **C2** while simultaneously equating the strain from **C1** with that of the Ramberg-Osgood equation **C2**

$$\frac{S_{0.70E}}{(0.70\cdot E)} = \frac{S_{0.70E}}{E} + e_{0.2}\cdot\left(\frac{S_{0.70E}}{S_{0.2}}\right)^n$$

$$\frac{S_{0.70E}}{(0.70 \cdot E)} = \frac{S_{0.70E}}{E} + e_{0.2} \cdot \left(\frac{S_{0.70E}}{S_{0.2}}\right)^{n}$$

$$\left(\frac{1}{0.70} - 1\right) \cdot \frac{S_{0.70E}}{E} = e_{0.2} \cdot \left(\frac{S_{0.70E}}{S_{0.2}}\right)^{n}$$

$$\left(\frac{3}{7}\right) \cdot \frac{S_{0.70E}}{E} = e_{0.2} \cdot \left(\frac{S_{0.70E}}{S_{0.2}}\right)^{n}$$

Since $n > 1$, then divide through by $\left(S_{0.70E}\right) \cdot \left(e_{0.2}\right)$

$$\left(\frac{3}{7 \cdot e_{0.2}}\right) \cdot \frac{1}{E} = \frac{\left(S_{0.70E}\right)^{(n-1)}}{S_{0.2}{}^{n}}$$

Solve for $S_{0.70E}$:

$$\boxed{S_{0.70E} = \left[\frac{S_{0.2}}{\left[\left(\frac{7 \cdot e_{0.2}}{3}\right) \cdot E\right]^{\frac{1}{n}}}\right]^{\left(\frac{n}{n-1}\right)}}$$

Recall that $e_{0.2} = 0.2 \cdot \%$ therefore, $7 \cdot e_{0.2} = 0.014$, that is

$$\boxed{S_{0.70E} = \left[\frac{S_{0.2}}{\left[\left(\frac{0.014}{3}\right) \cdot E\right]^{\frac{1}{n}}}\right]^{\left(\frac{n}{n-1}\right)}} \qquad (3)$$

Determine the strain, at which the 0.70xE line, **C1**, intercepts the Ramberg-Osgood line, **C2**

$$\varepsilon_{70E} = \frac{(0.70 \cdot E) \cdot \varepsilon_{70E}}{E} + e_{0.2} \cdot \left[\frac{(0.70 \cdot E) \cdot \varepsilon_{70E}}{S_{0.2}}\right]^n$$

$$0.30 \cdot \varepsilon_{70E} = e_{0.2} \cdot \left[\frac{(0.70 \cdot E) \cdot \varepsilon_{70E}}{S_{0.2}}\right]^n$$

Divide through by $(\varepsilon_{70E}) \cdot (e_{0.2})$ $\quad \frac{0.30}{e_{0.2}} = \left(\frac{0.70 \cdot E}{S_{0.2}}\right)^n \cdot \varepsilon_{70E}^{(n-1)}$

Solve for ε_{70E} $\quad \varepsilon_{70E} = \left[\left(\frac{0.30}{e_{0.2}}\right) \cdot \left(\frac{S_{0.2}}{0.70 \cdot E}\right)^n\right]^{\left(\frac{1}{n-1}\right)}$

Again, $e_{0.2} = 0.2 \cdot \%$ therefore, $\frac{0.30}{e_{0.2}} = 150$, that is

$$\boxed{\varepsilon_{70E} = \left[150 \cdot \left(\frac{S_{0.2}}{0.70 \cdot E}\right)^n\right]^{\left(\frac{1}{n-1}\right)}} \qquad (4)$$

The procedure for obtaining the explicit $S_{0.70E}$ from a cubic-spline stress-strain data set, or a stress-strain model such as equation **C2** (Ramberg-Osgood) is as follows:

Step 1: Solve ε_{70E} for the given the yield strength, $S_{0.2} = F_{ty}$, the modulus of elasticity, E, and the exponent, n

Step 2: $S_{0.70E} = F_G(\varepsilon_{70E})$

Where the subscript of the function may be either RO, for Ramberg-Osgood, or CS for Cubic-spline interpolation function

It should be noted that if **step 1** is not followed, then a root-finding procedure must be utilized to numerically find the interception point, $\varepsilon = \varepsilon_{70E}$

Appendix D:
Stress Concentration Factor (SCF) of a plate in tension with an eccentric hole.

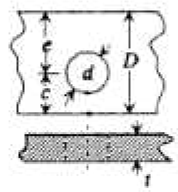


Figure 1: Eccentric circular hole in finite-width plane

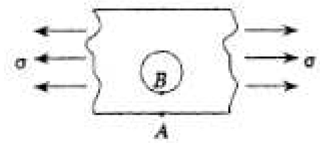


Figure 2: Axial tension

Yield-stress Margin of Safety: $\boxed{MS_y = \frac{F_{ty}}{\sigma_{nom}} - 1}$

Ultimate-stress Margin of Safety: $\boxed{MS_y = \frac{F_{tu}}{1.50 \cdot \sigma_{nom}} - 1}$ or $\boxed{MS_y = \frac{F_{tu}}{\sigma_{ult}} - 1}$

Where $\sigma_{nom} = \Lambda(\lambda, \psi) \cdot \sigma_g = \dfrac{\int_B^A \sigma(y)\, dy}{A - B}$ $\quad \lambda = \dfrac{d}{2 \cdot c}$ and $\psi = \dfrac{e}{c}$

Peterson's Function (Reference 1): $\boxed{\alpha := (3.000 \;\; -3.140 \;\; 3.667 \;\; -1.527)^T}$

$$\boxed{K_n(\lambda) := \alpha_1 + \sum_{i=2}^{4} \left(\alpha_i \cdot \lambda^{i-1}\right)} \qquad (1)$$

$$\boxed{\Lambda(\lambda, \psi) \equiv \frac{\sqrt{1 - \lambda^2}}{(1 - \lambda)} \cdot \left[\frac{1}{1 - \frac{1}{\psi} \cdot \left(1 - \sqrt{1 - \lambda^2}\right)}\right]} \qquad (2)$$

$$\boxed{K_t(\lambda, \psi) = \Lambda(\lambda, \psi) \cdot K'_{tn}(\lambda, \psi)} \qquad (3)$$

Equation (3) is implied in Chart 4.3 or reference 3.

Chart 4.3 from Reference 3

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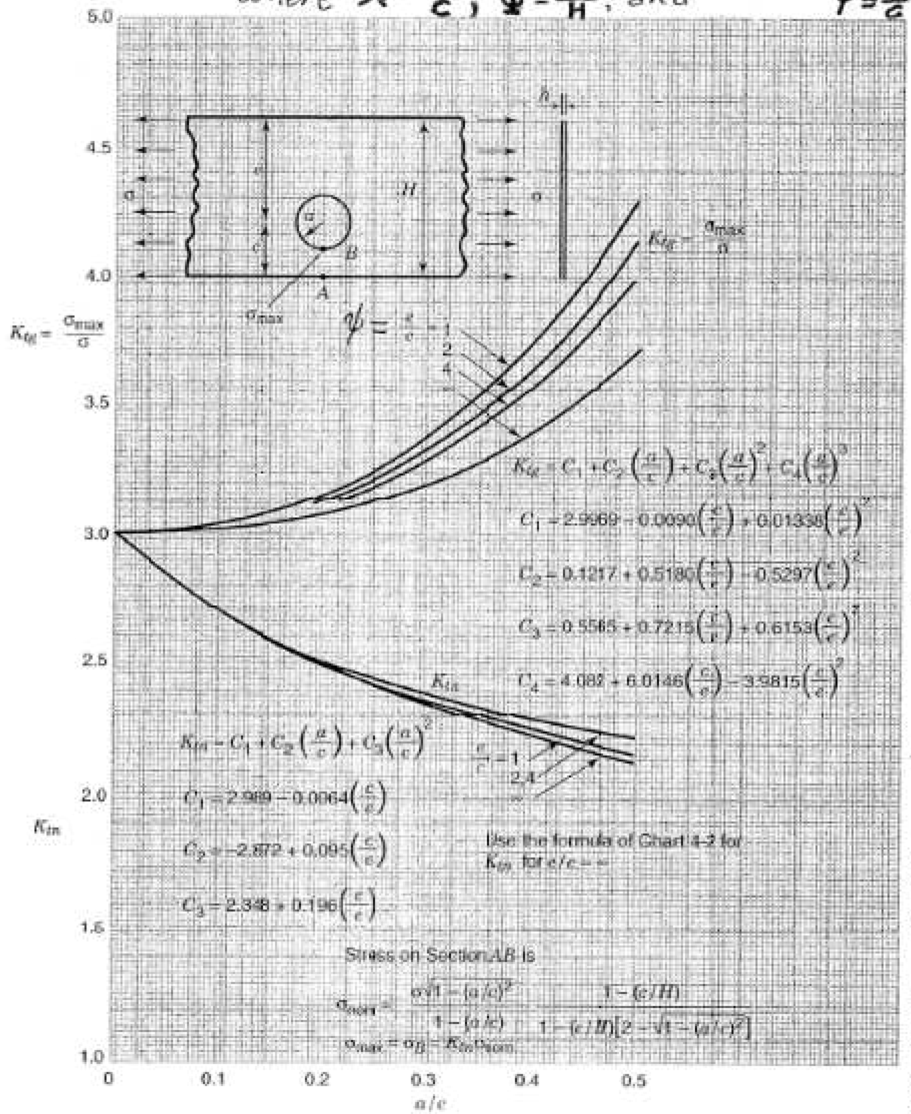
$\sigma_{nom} = \Lambda'(\lambda, \Phi) \cdot \sigma = \Lambda(\lambda, \psi) \cdot \sigma$ *

where $\lambda = \frac{a}{c}$, $\Phi = \frac{c}{H}$, and $\psi = \frac{e}{c}$

$\sigma_{max} = K_{tn} \cdot \sigma_{nom}$

$K_{tn} = 3.000 - 3.140 \cdot \lambda + 3.667 \cdot \lambda^2 - 1.527 \cdot \lambda^3$ **



** Walter D. Pilkey, "Formulas for Stress, strain, and Structural Matrices", Second Edition, Page 285

Chart 4.3 Stress concentration factors for the tension of a finite-width element having an eccentrically located circular hole (based on mathematical analysis of Sjöström 1950). $e/c = \infty$ corresponds to Chart 4.2.

$$* \quad \Lambda(\lambda, \psi) = \frac{\sqrt{1-\lambda^2}}{1-\lambda} \cdot \left[\frac{1}{1 - \frac{1}{\psi}\sqrt{1-\lambda^2}}\right]$$, (See pages 184-185)

References

1. Walter D. Pilkey, "Formulas for Stress, Strain and Structural Matrices", Second Edition. ISBN 0-471-03221-2, Pages 285, and 330 Through 332.
2. Rudolph Earl Peterson, "Stress Concentration Factors", First Edition. ISBN 9780471683292, Page 152.
3. Walter D. Pilkey and Deborah F. Pilkey "Peterson's Stress Concentration Factors", Third Edition.

ORIGIN = 1 $\qquad \boxed{\psi' = \frac{1}{\psi} = \frac{C}{e}} \qquad \psi = 1.504 \qquad \psi' := \frac{1}{\psi} = 0.665$

Gross SCF Function

$$c_g(\psi) \equiv \begin{bmatrix} \left(2.9969 - \frac{0.0090}{\psi} + \frac{0.01338}{\psi^2}\right) \\ \left(0.1217 + \frac{0.5180}{\psi} - \frac{0.5297}{\psi^2}\right) \\ \left(0.5565 + \frac{0.7215}{\psi} + \frac{0.6153}{\psi^2}\right) \\ \left(4.0482 + \frac{6.0146}{\psi} - \frac{3.9815}{\psi^2}\right) \end{bmatrix} \qquad c_g(\psi) = \begin{pmatrix} 2.997 \\ 0.232 \\ 1.309 \\ 6.287 \end{pmatrix}$$

$$K'_{tg}(\lambda,\psi) \equiv c_g(\psi)_1 + c_g(\psi)_2 \cdot \lambda + c_g(\psi)_3 \cdot \lambda^2 + c_g(\psi)_4 \cdot \lambda^3$$

Nominal SCF Function

$$c_n(\psi) \equiv \begin{bmatrix} \left(2.989 - \frac{0.0064}{\psi}\right) \\ \left(-2.872 + \frac{0.095}{\psi}\right) \\ \left(2.348 + \frac{0.196}{\psi}\right) \end{bmatrix} \qquad c_n(\psi) = \begin{pmatrix} 2.985 \\ -2.809 \\ 2.478 \end{pmatrix} \qquad c_n(\Psi) = \begin{pmatrix} 2.986 \\ -2.825 \\ 2.446 \end{pmatrix}$$

$$K'_{tn}(\lambda,\psi) \equiv c_n(\psi)_1 + c_n(\psi)_2 \cdot \lambda + c_n(\psi)_3 \cdot \lambda^2$$

Determine the relationship between $K_t(\lambda,\psi)$ and $K'_{tn}(\lambda,\psi)$

$$\sigma_{max} = K'_{tn}(\lambda,\psi)\cdot\sigma_{Nom} \qquad (4)$$

$$\sigma_{Nom} = \Lambda(\lambda,\psi)\cdot\sigma_g \qquad (5)$$

Substitute equation (5) into (4):

Therefore, $\sigma_{max} = K'_{tn}(\lambda,\psi)\cdot\left(\Lambda(\lambda,\psi)\cdot\sigma_g\right)$

$$\sigma_{max} = K_t(\lambda,\psi)\cdot\sigma_g \qquad (6)$$

Where $\boxed{K_t(\lambda,\psi) = \Lambda(\lambda,\psi)\cdot K'_{tn}(\lambda,\psi)}$